

# Some aspects of multivariate Behrens-Fisher problem

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## Abstract

In this paper we discuss the well known multivariate Behrens-Fisher problem which deals with testing the equality of two normal mean vectors under heteroscedasticity of dispersion matrices. Some existing tests are reviewed and a new test based on Roy's union-intersection principle coupled with the generalized  $P$ -value is proposed. The tests are compared with respect to size and power based on simulation, and applied to a few useful data sets.

## Keywords

Behrens-Fisher problem, Generalized  $P$ -value, Heterogeneity, Union-intersection principle

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